Differentiation Technique - Chain Rule

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Question 1

Qualification: AP Calculus AB

Areas: Differentiation, Differential Equations

Subtopics: Differentiation Technique – Chain Rule, Differentiation Technique – Product Rule, Initial Conditions in Differential Equation, Separation of Variables in Differential Equation

Paper: Part B-Non-Calc / Series: 2003-Form-B / Difficulty: Easy / Question Number: 6

6. Let f be the function satisfying $f'(x) = x\sqrt{f(x)}$ for all real numbers x, where $f(3) = 25$.

   (a) Find $f''(3)$.

   (b) Write an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = x\sqrt{y}$ with the initial condition $f(3) = 25$.

Question 2

Qualification: AP Calculus AB

Areas: Differentiation, Applications of Differentiation

Subtopics: Implicit Differentiation, Tangents To Curves, Differentiation Technique – Chain Rule

Paper: Part B-Non-Calc / Series: 2005-Form-B / Difficulty: Somewhat Challenging / Question Number: 5

5. Consider the curve given by $y^2 = 2 + xy$.

(a) Show that $\frac{dy}{dx} = \frac{y}{2y - x}$.

(b) Find all points (x, y) on the curve where the line tangent to the curve has slope $\frac{1}{2}$.

(c) Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

(d) Let x and y be functions of time t that are related by the equation $y^2 = 2 + xy$. At time $t = 5$, the value of y is 3 and $\frac{dy}{dt} = 6$. Find the value of $\frac{dx}{dt}$ at time $t = 5$.

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Question 3

Qualification: AP Calculus AB

Areas: Differential Equations, Applications of Differentiation, Differentiation

Subtopics: Kinematics (Displacement, Velocity, and Acceleration), Global or Absolute Minima and Maxima, Differentiation Technique – Chain Rule, Differentiation Technique – Trigonometry, Differentiation Technique – Exponentials, Verifying Solutions to Differential Equation

Paper: Part B-Non-Calc / Series: 2007 / Difficulty: Somewhat Challenging / Question Number: 4

4. A particle moves along the x-axis with position at time t given by $x(t) = e^{-t} \sin t$ for $0 \le t \le 2\pi$.

   (a) Find the time t at which the particle is farthest to the left. Justify your answer.

   (b) Find the value of the constant A for which $x(t)$ satisfies the equation $Ax''(t) + x'(t) + x(t) = 0$ for $0 < t < 2\pi$.

Question 4

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Rates of Change (Instantaneous), Rates of Change (Average), Differentiation Technique – Chain Rule, Interpreting Meaning in Applied Contexts, Modelling Situations

Paper: Part A-Calc / Series: 2007-Form-B / Difficulty: Medium / Question Number: 3

3. The wind chill is the temperature, in degrees Fahrenheit $(°F)$, a human feels based on the air temperature, in degrees Fahrenheit, and the wind velocity v, in miles per hour (mph). If the air temperature is $32°F$, then the wind chill is given by $W(v) = 55.6 - 22.1v^{0.16}$ and is valid for $5 \le v \le 60$.

(a) Find $W'(20)$. Using correct units, explain the meaning of $W'(20)$ in terms of the wind chill.

(b) Find the average rate of change of W over the interval $5 \le v \le 60$. Find the value of v at which the instantaneous rate of change of W is equal to the average rate of change of W over the interval $5 \le v \le 60$.

(c) Over the time interval $0 \le t \le 4$ hours, the air temperature is a constant $32°F$. At time $t = 0$, the wind velocity is $v = 20$ mph. If the wind velocity increases at a constant rate of 5 mph per hour, what is the rate of change of the wind chill with respect to time at $t = 3$ hours? Indicate units of measure.

Question 5

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Mean Value Theorem, Differentiation Technique – Chain Rule, Points Of Inflection, Intermediate Value Theorem

Paper: Part B-Non-Calc / Series: 2007-Form-B / Difficulty: Hard / Question Number: 6

6. Let f be a twice-differentiable function such that $f(2) = 5$ and $f(5) = 2$. Let g be the function given by $g(x) = f(f(x))$.

(a) Explain why there must be a value c for $2 < c < 5$ such that $f'(c) = -1$.

(b) Show that $g'(2) = g'(5)$. Use this result to explain why there must be a value k for $2 < k < 5$ such that $g''(k) = 0$.

(c) Show that if $f''(x) = 0$ for all x, then the graph of g does not have a point of inflection.

(d) Let $h(x) = f(x) - x$. Explain why there must be a value r for $2 < r < 5$ such that $h(r) = 0$.

Question 6

Qualification: AP Calculus AB

Areas: Applications of Integration, Differentiation

Subtopics: Kinematics (Displacement, Velocity, and Acceleration), Rates of Change (Instantaneous), Differentiation Technique – Chain Rule, Total Amount

Paper: Part A-Calc / Series: 2008-Form-B / Difficulty: Medium / Question Number: 2

2. For time $t \geq 0$ hours, let $r(t) = 120\left(1 - e^{-10t^2}\right)$ represent the speed, in kilometers per hour, at which a car travels along a straight road. The number of liters of gasoline used by the car to travel x kilometers is modeled by $g(x) = 0.05x\left(1 - e^{-x/2}\right)$.

(a) How many kilometers does the car travel during the first 2 hours?

(b) Find the rate of change with respect to time of the number of liters of gasoline used by the car when $t = 2$ hours. Indicate units of measure.

(c) How many liters of gasoline have been used by the car when it reaches a speed of 80 kilometers per hour?

Question 7

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Fundamental Theorem of Calculus (Second), Differentiation Technique – Chain Rule, Differentiation Technique – Trigonometry, Tangents To Curves, Global or Absolute Minima and Maxima

Paper: Part B-Non-Calc / Series: 2008-Form-B / Difficulty: Easy / Question Number: 4

4. The functions f and g are given by $f(x) = \int_0^{3x} \sqrt{4 + t^2}\, dt$ and $g(x) = f(\sin x)$.

(a) Find $f'(x)$ and $g'(x)$.

(b) Write an equation for the line tangent to the graph of $y = g(x)$ at $x = \pi$.

(c) Write, but do not evaluate, an integral expression that represents the maximum value of g on the interval $0 \le x \le \pi$. Justify your answer.

Question 8

Qualification: AP Calculus AB

Areas: Limits and Continuity, Applications of Differentiation, Differentiation, Integration

Subtopics: Differentiation Technique – Chain Rule, Differentiation Technique – Standard Functions, Tangents To Curves, Continuities and Discontinuities, Integration Technique – Substitution

Paper: Part B-Non-Calc / Series: 2012 / Difficulty: Medium / Question Number: 4

4. The function f is defined by $f(x) = \sqrt{25 - x^2}$ for $-5 \le x \le 5$.

(a) Find $f'(x)$.

(b) Write an equation for the line tangent to the graph of f at $x = -3$.

(c) Let g be the function defined by $g(x) = \begin{cases} f(x) & \text{for } -5 \le x \le -3 \\ x + 7 & \text{for } -3 < x \le 5. \end{cases}$

Is g continuous at $x = -3$? Use the definition of continuity to explain your answer.

(d) Find the value of $\int_0^5 x\sqrt{25 - x^2}\, dx$.

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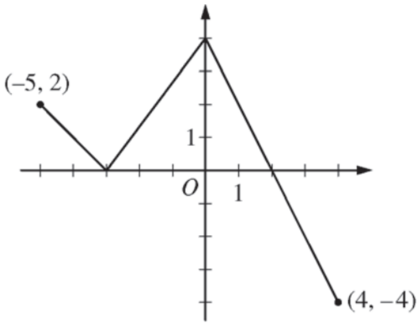
Question 9

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation, Integration

Subtopics: Integration Technique – Geometric Areas, Increasing/Decreasing , Concavity, Differentiation Technique - Quotient Rule, Tangents To Curves, Differentiation Technique – Chain Rule, Integration Graphs

Paper: Part B-Non-Calc / Series: 2014 / Difficulty: Medium / Question Number: 3



Graph of f

3. The function f is defined on the closed interval $[-5, 4]$. The graph of f consists of three line segments and is shown in the figure above. Let g be the function defined by $g(x) = \int_{-3}^{x} f(t)\,dt$.

(a) Find $g(3)$.

(b) On what open intervals contained in $-5 < x < 4$ is the graph of g both increasing and concave down? Give a reason for your answer.

(c) The function h is defined by $h(x) = \dfrac{g(x)}{5x}$. Find $h'(3)$.

(d) The function p is defined by $p(x) = f\left(x^2 - x\right)$. Find the slope of the line tangent to the graph of p at the point where $x = -1$.

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Question 10

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Mean Value Theorem, Local or Relative Minima and Maxima, Derivative Tables, Differentiation Technique – Chain Rule, Fundamental Theorem of Calculus (First)

Paper: Part B-Non-Calc / Series: 2014 / Difficulty: Somewhat Challenging / Question Number: 5

x	-2	$-2 < x < -1$	-1	$-1 < x < 1$	1	$1 < x < 3$	3
$f(x)$	12	Positive	8	Positive	2	Positive	7
$f'(x)$	-5	Negative	0	Negative	0	Positive	$\frac{1}{2}$
$g(x)$	-1	Negative	0	Positive	3	Positive	1
$g'(x)$	2	Positive	$\frac{3}{2}$	Positive	0	Negative	-2

5. The twice-differentiable functions f and g are defined for all real numbers x. Values of f, f', g, and g' for various values of x are given in the table above.

(a) Find the x-coordinate of each relative minimum of f on the interval $[-2, 3]$. Justify your answers.

(b) Explain why there must be a value c, for $-1 < c < 1$, such that $f''(c) = 0$.

(c) The function h is defined by $h(x) = \ln(f(x))$. Find $h'(3)$. Show the computations that lead to your answer.

(d) Evaluate $\int_{-2}^{3} f'(g(x))g'(x)\,dx$.

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Question 11

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Derivative Tables, Tangents To Curves, Differentiation Technique - Quotient Rule, Fundamental Theorem of Calculus (First), Differentiation Technique – Chain Rule

Paper: Part B-Non-Calc / Series: 2016 / Difficulty: Medium / Question Number: 6

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	–6	3	2	8
2	2	–2	–3	0
3	8	7	6	2
6	4	5	3	–1

6. The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x.

(a) Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.

(b) Let $h(x) = \dfrac{g(x)}{f(x)}$. Find $h'(1)$.

(c) Evaluate $\int_1^3 f''(2x)\, dx$.

Question 12

Qualification: AP Calculus AB

Areas: Applications of Integration, Integration

Subtopics: Riemann Sums – Left , Increasing/Decreasing , Modelling Situations, Differentiation Technique – Chain Rule, Related Rates

Paper: Part A-Calc / Series: 2017 / Difficulty: Medium / Question Number: 1

h (feet)	0	2	5	10
$A(h)$ (square feet)	50.3	14.4	6.5	2.9

1. A tank has a height of 10 feet. The area of the horizontal cross section of the tank at height h feet is given by the function A, where $A(h)$ is measured in square feet. The function A is continuous and decreases as h increases. Selected values for $A(h)$ are given in the table above.

(a) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the volume of the tank. Indicate units of measure.

(b) Does the approximation in part (a) overestimate or underestimate the volume of the tank? Explain your reasoning.

(c) The area, in square feet, of the horizontal cross section at height h feet is modeled by the function f given by $f(h) = \dfrac{50.3}{e^{0.2h} + h}$. Based on this model, find the volume of the tank. Indicate units of measure.

(d) Water is pumped into the tank. When the height of the water is 5 feet, the height is increasing at the rate of 0.26 foot per minute. Using the model from part (c), find the rate at which the volume of water is changing with respect to time when the height of the water is 5 feet. Indicate units of measure.

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Question 13

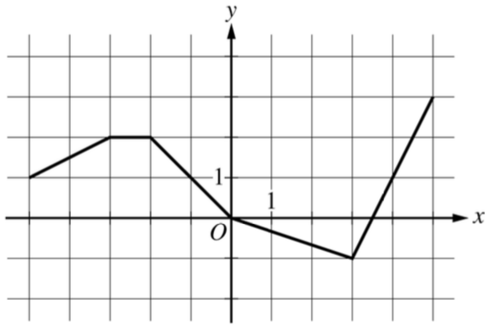
Qualification: AP Calculus AB

Areas: Differentiation, Applications of Differentiation

Subtopics: Derivative Tables, Tangents To Curves, Differentiation Technique – Chain Rule, Derivative Graphs, Differentiation Technique – Product Rule, Mean Value Theorem, Differentiation Technique – Trigonometry, Differentiation Technique – Exponentials

Paper: Part B-Non-Calc / Series: 2017 / Difficulty: Medium / Question Number: 6

x	$g(x)$	$g'(x)$
–5	10	–3
–4	5	–1
–3	2	4
–2	3	1
–1	1	–2
0	0	–3



Graph of h

6. Let f be the function defined by $f(x) = \cos(2x) + e^{\sin x}$.

Let g be a differentiable function. The table above gives values of g and its derivative g' at selected values of x.

Let h be the function whose graph, consisting of five line segments, is shown in the figure above.

(a) Find the slope of the line tangent to the graph of f at $x = \pi$.

(b) Let k be the function defined by $k(x) = h(f(x))$. Find $k'(\pi)$.

(c) Let m be the function defined by $m(x) = g(-2x) \cdot h(x)$. Find $m'(2)$.

(d) Is there a number c in the closed interval $[-5, -3]$ such that $g'(c) = -4$? Justify your answer.

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Question 14

Qualification: AP Calculus AB

Areas: Limits and Continuity, Differentiation

Subtopics: Differentiation Technique – Product Rule, L'Hôpital's Rule, Calculating Limits Algebraically, Differentiation Technique – Chain Rule, Continuities and Discontinuities

Paper: Part B-Non-Calc / Series: 2019 / Difficulty: Very Hard / Question Number: 6

6. Functions f, g, and h are twice-differentiable functions with $g(2) = h(2) = 4$. The line $y = 4 + \frac{2}{3}(x - 2)$ is tangent to both the graph of g at $x = 2$ and the graph of h at $x = 2$.

(a) Find $h'(2)$.

(b) Let a be the function given by $a(x) = 3x^3 h(x)$. Write an expression for $a'(x)$. Find $a'(2)$.

(c) The function h satisfies $h(x) = \dfrac{x^2 - 4}{1 - (f(x))^3}$ for $x \neq 2$. It is known that $\lim\limits_{x \to 2} h(x)$ can be evaluated using L'Hospital's Rule. Use $\lim\limits_{x \to 2} h(x)$ to find $f(2)$ and $f'(2)$. Show the work that leads to your answers.

(d) It is known that $g(x) \leq h(x)$ for $1 < x < 3$. Let k be a function satisfying $g(x) \leq k(x) \leq h(x)$ for $1 < x < 3$. Is k continuous at $x = 2$? Justify your answer.

Question 15

Qualification: AP Calculus AB

Areas: Applications of Integration, Applications of Differentiation, Integration

Subtopics: Interpreting Meaning in Applied Contexts, Rates of Change (Average), Riemann Sums – Right, Increasing/Decreasing , Differentiation Technique – Chain Rule, Average Value of a Function

Paper: Part A-Calc / Series: 2021 / Difficulty: Somewhat Challenging / Question Number: 1

r (centimeters)	0	1	2	2.5	4
$f(r)$ (milligrams per square centimeter)	1	2	6	10	18

1. The density of a bacteria population in a circular petri dish at a distance r centimeters from the center of the dish is given by an increasing, differentiable function f, where $f(r)$ is measured in milligrams per square centimeter. Values of $f(r)$ for selected values of r are given in the table above.

(a) Use the data in the table to estimate $f'(2.25)$. Using correct units, interpret the meaning of your answer in the context of this problem.

(b) The total mass, in milligrams, of bacteria in the petri dish is given by the integral expression $2\pi \int_0^4 r f(r)\, dr$. Approximate the value of $2\pi \int_0^4 r f(r)\, dr$ using a right Riemann sum with the four subintervals indicated by the data in the table.

(c) Is the approximation found in part (b) an overestimate or underestimate of the total mass of bacteria in the petri dish? Explain your reasoning.

(d) The density of bacteria in the petri dish, for $1 \le r \le 4$, is modeled by the function g defined by $g(r) = 2 - 16(\cos(1.57\sqrt{r}))^3$. For what value of k, $1 < k < 4$, is $g(k)$ equal to the average value of $g(r)$ on the interval $1 \le r \le 4$?

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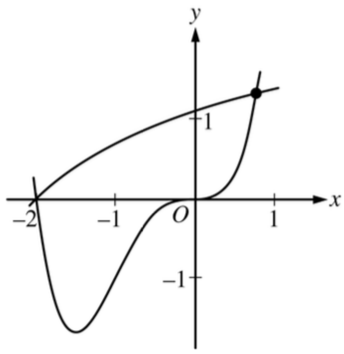
Question 16

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Applications of Integration

Subtopics: Integration - Area Between Curves, Increasing/Decreasing , Volume using Cross Sections, Rates of Change (Instantaneous), Differentiation Technique – Chain Rule

Paper: Part A-Calc / Series: 2022 / Difficulty: Medium / Question Number: 2



2. Let f and g be the functions defined by $f(x) = \ln(x+3)$ and $g(x) = x^4 + 2x^3$. The graphs of f and g, shown in the figure above, intersect at $x = -2$ and $x = B$, where $B > 0$.

(a) Find the area of the region enclosed by the graphs of f and g.

(b) For $-2 \leq x \leq B$, let $h(x)$ be the vertical distance between the graphs of f and g. Is h increasing or decreasing at $x = -0.5$? Give a reason for your answer.

(c) The region enclosed by the graphs of f and g is the base of a solid. Cross sections of the solid taken perpendicular to the x-axis are squares. Find the volume of the solid.

(d) A vertical line in the xy-plane travels from left to right along the base of the solid described in part (c). The vertical line is moving at a constant rate of 7 units per second. Find the rate of change of the area of the cross section above the vertical line with respect to time when the vertical line is at position $x = -0.5$.

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Question 17

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation, Integration

Subtopics: Rates of Change (Average), Intermediate Value Theorem, Riemann Sums – Right, Implicit Differentiation, Rates of Change (Instantaneous), Differentiation Technique – Chain Rule, Related Rates

Paper: Part B-Non-Calc / Series: 2022 / Difficulty: Medium / Question Number: 4

t (days)	0	3	7	10	12
$r'(t)$ (centimeters per day)	-6.1	-5.0	-4.4	-3.8	-3.5

4. An ice sculpture melts in such a way that it can be modeled as a cone that maintains a conical shape as it decreases in size. The radius of the base of the cone is given by a twice-differentiable function r, where $r(t)$ is measured in centimeters and t is measured in days. The table above gives selected values of $r'(t)$, the rate of change of the radius, over the time interval $0 \le t \le 12$.

(a) Approximate $r''(8.5)$ using the average rate of change of r' over the interval $7 \le t \le 10$. Show the computations that lead to your answer, and indicate units of measure.

(b) Is there a time t, $0 \le t \le 3$, for which $r'(t) = -6$? Justify your answer.

(c) Use a right Riemann sum with the four subintervals indicated in the table to approximate the value of $\int_0^{12} r'(t)\, dt$.

(d) The height of the cone decreases at a rate of 2 centimeters per day. At time $t = 3$ days, the radius is 100 centimeters and the height is 50 centimeters. Find the rate of change of the volume of the cone with respect to time, in cubic centimeters per day, at time $t = 3$ days. (The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.)

Question 18

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Derivative Tables, Differentiation Technique – Chain Rule, Concavity, Differentiation Technique – Product Rule, Fundamental Theorem of Calculus (First), Increasing/Decreasing

Paper: Part B-Non-Calc / Series: 2023 / Difficulty: Somewhat Challenging / Question Number: 5

x	0	2	4	7
$f(x)$	10	7	4	5
$f'(x)$	$\frac{3}{2}$	-8	3	6
$g(x)$	1	2	-3	0
$g'(x)$	5	4	2	8

5. The functions f and g are twice differentiable. The table shown gives values of the functions and their first derivatives at selected values of x.

(a) Let h be the function defined by $h(x) = f(g(x))$. Find $h'(7)$. Show the work that leads to your answer.

(b) Let k be a differentiable function such that $k'(x) = (f(x))^2 \cdot g(x)$. Is the graph of k concave up or concave down at the point where $x = 4$? Give a reason for your answer.

(c) Let m be the function defined by $m(x) = 5x^3 + \int_0^x f'(t)\, dt$. Find $m(2)$. Show the work that leads to your answer.

(d) Is the function m defined in part (c) increasing, decreasing, or neither at $x = 2$? Justify your answer.